Pan Pearl River Delta Physics Olympiad 2026
2026 年泛珠三角及中华名校物理奥林匹克邀请赛
Sponsored by Institute for Advanced Study, HKUST. 香港科技大学高等研究院赞助
Paper-1 (Total 4 Problems, 10 Points each) 卷-1（共4题，每题10分）(9:30 am – 12:00 pm, 21 Feb 2026)

Please fill in your **final answers** to all problems on the answer sheet.
请在答题纸上填上各题的**最后答案**。
At the end of the competition, please submit the answer sheet only. Question papers and working sheets will not be collected.
比赛结束时，请只交回答题纸，题目纸和草稿纸将不会收回。

1. Rotating Cylinder 旋转圆柱体

(a) [1] Alice shows Bob her new cylinder collection. One of them is a hollow cylinder. The cylinder has length L, radius r and wall thickness $\delta \ll r$ for both the bases and the curved surface. The mass density of the cylinder is ρ. Find the mass m of the cylinder and the moment of inertia I of the cylinder about its axis.

(a) [1] Alice 向 Bob 展示了她的新圆柱体收藏。其中一个是**空心圆柱体**。该圆柱体的长度为 L，半径为 r，底面和侧面的壁厚均为 $\delta \ll r$。圆柱体的质量密度为 ρ。求圆柱体的质量 m 以及圆柱体绕其轴线的转动惯量 I。

(b) [2] The cylinder is made of carbon nanotubes and has high tensile strength, $\sigma = \frac{F_{max}}{A}$, where F_{max} is the maximum tension force the cylinder can sustain without breaking and A is the cross-sectional area perpendicular to the direction the force is applied (see Fig. 1-1). By considering an element of the cylinder when spinning (refer to the top view of the spinning cylinder in Fig 1-2), estimate the maximum angular velocity ω_{max} at which the cylinder can spin before breaking. Express in terms of σ, ρ and r. Give a numerical value of ω_{max} for $L = 1m, r = 10cm, \delta = 50\mu m, \sigma = 20GPa, \rho = 1.3g/cm^3$. Ignore deformation. You may assume the sides break before the bases do.

(b) [2] 该圆柱体由纳米碳管制成，具有极高的抗拉强度 $\sigma = \frac{F_{max}}{A}$，其中 F_{max} 是圆柱体在断裂前能承受的最大拉力，A 是垂直于受力方向的截面积 (見图 1-1)。通过考虑旋转圆柱体的一个微元（参考旋转圆柱体的俯视图 1-2），估算圆柱体断裂前的最大角速度 ω_{max}。用σ, ρ, r 来表达。

给出以下数值：$L = 1$ m, $r = 10$ cm, $\delta = 50$ μm, $\sigma = 20$ GPa, $\rho = 1.3 \times 9$ g/cm^3，计算ω_{max}的数值。忽略形变。你可以假设侧面先于底面断裂。

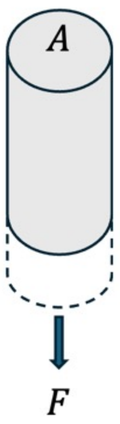
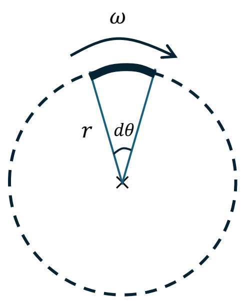


Fig 1-1



Top view
俯视图
Fig 1-2

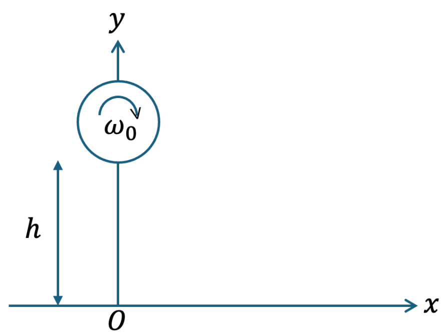


Fig 1-3

In what follows, we shall assume $\omega \ll \omega_{max}$.

(c) [2] In Fig 1-3, Alice uses a motor positioned at a height h above the ground to set the cylinder spinning with an initial angular velocity ω_0. In a prank, Bob uses his "psychic powers" to release the cylinder from the motor without altering its angular velocity. The cylinder, starting with a negligible initial translational velocity, falls toward the ground, which has a coefficient of friction μ. Ignoring air resistance, assume that collisions with the ground are perfectly elastic in the y-direction and that the collision duration t_c is negligible ($gt_c^2 \ll h$), where g represents the acceleration due to gravity. Sketch a qualitative graph representing the trajectory of the cylinder's center of mass.

在下文中，我们假设 $\omega \ll \omega_{max}$。

(c) [2] 在图 1-3，Alice 使用安装在距离地面高度为 h 处的电机，使圆柱体以初始角速度 ω_0旋转。随后 Bob 搞了个恶作剧，利用他的“超能力”使圆柱体脱离电机，且脱离过程不改变其角速度。在圆柱体的初始平动速度可忽略不计的情况下向地面下落，地面与圆柱体间的摩擦系数为 μ。忽略空气阻力，并假设：与地面的碰撞在 y 方向上为完全弹性碰撞，且碰撞时间 t_c 极短（满足 $gt_c^2 \ll h$），其中 g 为重力加速度。请画出圆柱体质心运动轨迹的定性图。

(d) The cylinder hits the ground at O at $t = 0$.
(di) [2] Find the velocity (v_x, v_y) and the angular velocity ω of the cylinder right after the 1st collision.
(dii) [3] Find the position of the ball $x(t)$ and $y(t)$, as well as the angular velocity $\omega(t)$. Express your answers in terms of the number of collisions with the ground N (We take $N = 1$ at $t = 0$), the time elapsed since the last collision Δt, h, g, r, and the ratio $K = \frac{mr^2}{I}$. You can use the floor function $\lfloor x \rfloor$ which denotes the integer part of x (e.g. $\lfloor \pi \rfloor = 3, \lfloor e \rfloor = 2$).

(d) 圆柱体在 $t = 0$ 时击中地面原点 O。

(di) [2] 求第一次碰撞后瞬间圆柱体的速度 (v_x, v_y)和角速度 ω。

(dii) [3] 求圆柱体的位置 $x(t)$和 $y(t)$，以及角速度 $\omega(t)$。用与地面碰撞的次数N（我们取 $t = 0$时 $N = 1$）、自上次碰撞以来经过的时间 Δt、h、g、r 以及比率 $K = \frac{mr^2}{I}$ 来表达你的答案。你可以使用下取整函数（floor function）$\lfloor x \rfloor$，它表示 x 的整数部分（例如 $\lfloor \pi \rfloor = 3, \lfloor e \rfloor = 2$）。

2. [10] Rods and Planes 棒与面

(a) Gravitational Field Generated by Rods 由细棒产生的引力场

(ai) [1] Find the magnitude of gravitational field at a distance d from an infinitely long thin rod with mass per unit length λ.
(aii) [3] Consider 2026 infinitely long thin rods oriented along the z axis, each with mass per unit length λ. The 2026 rods are on positions $(x_n, y_n) = \left(r_0 \cos\frac{2n\pi}{2026}, r_0 \sin\frac{2n\pi}{2026}\right)$, where $0 \le n \le 2025$. Find the net force on a point mass with mass m located at $(d, 0)$. (Hint: You can consider the complex roots of the polynomial: $(z + d)^{2026} - r_0^{2026} = 0$)
(aiii) [1] Suppose the small point mass is constrained to only move along the x axis, determine if it will oscillate around the origin. (Just answer yes or no)

(ai) [1] 求距离一根线质量密度为 λ 的“無限长细棒”距离 d 处的引力场强度。
(aii) [3] 考虑 2026 根沿 z 方向排列的無限长细棒，每根细棒的质量线密度皆为 λ。第 n 根细棒在平面上的位置为 $(x_n, y_n) = \left(r_0 \cos\frac{2n\pi}{2026}, r_0 \sin\frac{2n\pi}{2026}\right)$。求质量为 m 的质点在 $(d, 0)$ 点的合外力。
（提示：你可以考虑多项式$(z + d)^{2026} - r_0^{2026} = 0$ 的复根）
(aiii) [1] 若该质点仅被限制在 x 轴运动，判斷其会否在原点附近來回移动。（只需回答“是”或“否”）

(b) Gravitational Field Generated by a Plate. 由平板产生的引力场

(bi) [1] Consider an infinite horizontal plane with mass per unit area σ, find the magnitude of gravitational field a distance h above the plate.
(bii) [2] Consider a flat and horizontal circular plate with radius R and total mass m. Find the magnitude of gravitational field due to the plate at a distance $h = R$ above the center of the plate if the plate has uniform density.
(Hint: $\int_0^1 \frac{x\mathrm{d}x}{(1+x^2)^{\frac{3}{2}}} = 1 - \frac{\sqrt{2}}{2}$)
(biii) [2] Consider a horizontal, equilateral triangular plate of mass m, and side length a. Find the magnitude of gravitational field due to the plate at a distance $h = \frac{\sqrt{6}a}{12}$ above the centroid of the plate if the plate has uniform density. (Hint: The distance from the center of a regular tetrahedron to any of its face is $\frac{\sqrt{6}}{12}$ of the side length of each face.)
(bi) [1] 考虑一张无限大的水平平板，其面质量密度为 σ，求在距离平板高度为 h 处的引力场强度。
(bii) [2] 考虑一张水平放置的圆形平板，半径为 R，质量平均分佈，总质量为 m。求在距离平板中心高度$h = R$处，由该平板产生的引力场强度。（提示：$\int_0^1 \frac{x\mathrm{d}x}{(1+x^2)^{\frac{3}{2}}} = 1 - \frac{\sqrt{2}}{2}$）
(biii) [2] 考虑一个质量为 m、边长为 a的水平正三角形板。求位于平板质心正上方、高度 $h = \frac{\sqrt{6}a}{12}$ 处的引力场强度。（提示：正四面体的中心到任一面的距离为其边长的$\frac{\sqrt{6}}{12}$。）

3. Transmission Line Modelling 传输线模型

Consider a high-frequency signal propagating along a transmission line. The line is modeled as a series of infinitesimal segments of length Δx (See Fig 3-1). Let L and R be the series inductance and resistance per unit length, and G and C be the shunt conductance and capacitance per unit length, respectively. Assume L, R, G, and C are frequency-independent constants.
考虑高频信号在传输线中的传播。我们将传输线建模为长度为 Δx 的无穷小段的集合 (见图 3-1)。令 L 和 R 分别为单位长度的串联电感和电阻，G 和 C 分别为单位长度的并联电导和电容。假设 L, R, G, C 均为与频率无关的常数。

1

(ai) [4] By applying Kirchhoff's laws to the infinitesimal circuit model as shown in Fig. 3-1, derive the two coupled partial differential equations relating current $i(x,t)$ and voltage $v(x,t)$ as $\Delta x \to 0$.

(ai) [4] 对图 3-1 的电路模型应用基尔霍夫定律，推导出当 $\Delta x \to 0$ 时，描述电流 $i(x,t)$ 与电压 $v(x,t)$ 关系的两个耦合偏微分方程。

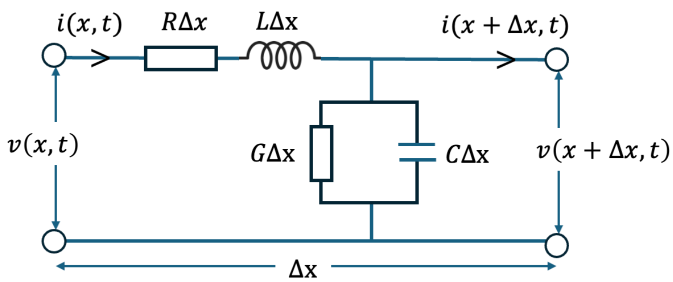


Fig 3-1

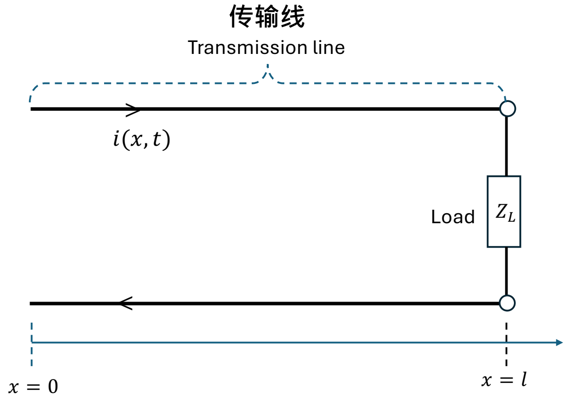


Fig 3-2

(aii) [1] For a sinusoidal signal of angular frequency ω, define $i(x,t) = \text{Re}(I(x)e^{j\omega t})$ and $v(x,t) = \text{Re}(V(x)e^{j\omega t})$, the equations reduce to:

$$\frac{d^2 I(x)}{dx^2} = \gamma^2 I(x), \quad \frac{d^2 V(x)}{dx^2} = \gamma^2 V(x)$$

Find the complex propagation constant γ^2 in terms of L, R, G, C, and ω. Here $j = \sqrt{-1}$ is the imaginary number and Re(z) is the real part of a complex number z.

(aii) [1] 对于角频率为 ω 的正弦信号，设 $i(x,t) = \text{Re}(I(x)e^{j\omega t})$ 且 $v(x,t) = \text{Re}(V(x)e^{j\omega t})$，方程可化简为：

$$\frac{d^2 I(x)}{dx^2} = \gamma^2 I(x), \quad \frac{d^2 V(x)}{dx^2} = \gamma^2 V(x)$$

求复传播常数 γ^2 关于 L, R, G, C 和 ω 的表达式。这里 $j = \sqrt{-1}$ 是虚数，$\text{Re}(z)$表示复数 z 的实部。

(aiii) [1] The general solutions are $I(x) = I_0^+ e^{-\gamma x} + I_0^- e^{\gamma x}$ and $V(x) = V_0^+ e^{-\gamma x} + V_0^- e^{\gamma x}$. Define the characteristic impedance $Z_0 = \left|\frac{V_0^+}{I_0^+}\right| = \left|\frac{V_0^-}{I_0^-}\right|$. Determine Z_0 in terms of L, R, G, C, and ω.

(aiii) [1] 已知通解为 $I(x) = I_0^+ e^{-\gamma x} + I_0^- e^{\gamma x}$ 和 $V(x) = V_0^+ e^{-\gamma x} + V_0^- e^{\gamma x}$。定义特性阻抗为 $Z_0 = \left|\frac{V_0^+}{I_0^+}\right| = \left|\frac{V_0^-}{I_0^-}\right|$。求 Z_0 的表达式，用L, R, G, C 和 ω 表达。

(aiv) [2] Find the condition relating L, R, G and C that allows a signal to propagate without distortion. Determine the phase velocity v_p in this case.

(aiv) [2] 试求当 L, R, G, C 满足何种关系时，信号可实现无失真传播？并求出该情形下的相速度 v_p。

(bi) [1] In Fig. 3-2, a transmission line is terminated by a load impedance Z_L. When an incident signal V_i travels along the line, a portion is reflected as V_r. Derive the voltage reflection coefficient, $\Gamma = \frac{V_r}{V_i}$, in terms of Z_L and the characteristic impedance Z_0.

(bi) [1] 如图 3-2 所示，传输线终端接有负载阻抗 Z_L。当入射信号 V_i 沿传输线传播时，部分信号将被反射，形成反射波 V_r。试推导电压反射系数 $\Gamma = \frac{V_r}{V_i}$ 的表达式（用 Z_L 和特性阻抗 Z_0 表示）。

(bii) [1] State the required condition for the incident voltage signal V_i to undergo a π phase change upon reflection.

(bii) [1] 若要使入射电压信号 V_i 在反射时产生 π 的相位变化，负载阻抗 Z_L 需满足什么条件？

4. **The Thermodynamic Helmholtz Resonator** 热力学亥姆霍兹共振器

(a) [1] Determine the fundamental frequency of a sound wave resonating in a cylindrical pipe of length H that is **closed at one end and open at the other,** as shown in Fig. 4-1. Given that the speed of sound is $v = 343\ \mathrm{m/s}$ and $H = 30\ cm$, calculate the fundamental frequency f.

(a) [1] 确定在一个长度为 H、一端封闭另一端开口的圆柱形管中（如图 4-1 所示）产生声波共振的基频。已知声速 $v = 343\ \mathrm{m/s}$，管长 $H = 30\ \mathrm{cm}$，计算该基频 f。

(b) However, the result from part (a) deviates significantly from actual measurements. Therefore, a more accurate model is required. We model the bottle as a Helmholtz resonator (see Fig. 4-2). A Helmholtz resonator consists of a cavity of volume V connected to the open atmosphere (pressure P_0) by a cylindrical neck of length L and radius r. The air in the neck acts as a piston of mass m, causing adiabatic compression and expansion of the air inside the cavity. The adiabatic speed of sound is given by $v = \sqrt{\frac{\gamma RT}{M}}$, where M is the molar mass of air, γ is the adiabatic index, R is the ideal gas constant, and T is the temperature.Starting from the adiabatic condition for the air in the cavity, we can show that for small volume changes ΔV, the pressure change ΔP is related to the displacement x of the air plug by $\Delta P = -kx$. Hence, the equation of motion for the air in the neck can be derived and we can show that the air column will oscillate. Assume the mass of the air in the neck is $m = \rho_0 \pi r^2 L$, where ρ_0 is the equilibrium air density.

Fig 4-1

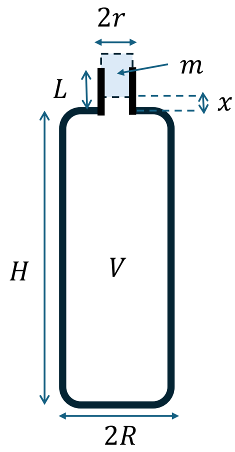


Fig 4-2

(b) 然而，(a) 部分的结果与瓶子的实际测量值偏差较大。因此，我们需要一个更精确的模型来描述瓶子产生的声波。我们将瓶子视为一个亥姆霍兹共振器（Helmholtz resonator，见图 4-2）。亥姆霍兹共振器由体积为 V 的空腔通过长为 L、半径为 r 的圆柱形颈管与大气（压强 P_0）相连而成。腔内空气经历绝热压缩和膨胀，而颈管内的空气则视作质量为 m 的活塞。绝热声速由 $v = \sqrt{\frac{\gamma RT}{M}}$ 给出，其中 M 为空气摩尔质量，γ 为绝热指数，R 为理想气体常数，T 为温度。从腔内空气的绝热条件出发，可证明对于微小的体积变化 ΔV，腔内压强变化 ΔP 与颈管内空气柱的位移 x 满足关系：$\Delta P = -kx$。由此，可推导颈管内空气的运动方程，並试证明颈管内的空气将作振荡运动。设颈管内空气质量为 $m = \rho_0 \pi r^2 L$，其中 ρ_0 为空气的平衡密度。

(bi) [5] Find the frequency f of the oscillating air. Express the answer in terms of P_0, ρ_0, r, L, V and γ.

(bi) [5] 求振动空气的频率 f。请用 P_0, ρ_0, r, L, V 和 γ 来表达你的答案。

(bii) [1] In reality, the length L is replaced by the effective length $L_{\mathrm{eff}} \approx L + 1.7r$, accounting for the air mass outside the neck that also oscillates. Calculate the fundamental frequency produced by the bottle with the following parameters: $H = 23\ \mathrm{cm}, \mathrm{R} = 3.5\ \mathrm{cm},\ L = 7\ \mathrm{cm}, r = 1\ \mathrm{cm}$.

(bii) [1] 在现实中，长度 L 会被有效长度 $L_{eff} \approx L + 1.7r$ 所取代，以考虑瓶颈外部同样发生振动的空气质量。计算具有以下参数的水瓶所产生的基频：$H = 23\ \mathrm{cm}$，$R = 3.5\ \mathrm{cm}$，$L = 7\ \mathrm{cm}$，$r = 1\ \mathrm{cm}$。

(c) The water bottle is constructed from a material with a linear thermal expansion coefficient α. Suppose the resonator is calibrated to a frequency f_0 at a reference temperature T_0. When the ambient temperature increases to $T = T_0 + \Delta T$ (where $\Delta T \ll T_0$):

(ci) [2] Account for both the change in the speed of sound of the air and the thermal expansion of the bottle's dimensions. Derive an expression for the new resonance frequency f in terms of $f_0, \Delta T, T_0$ and α.

(cii) [1] Determine the "critical" thermal expansion coefficient α_c at which the frequency of the bottle remains constant regardless of small temperature fluctuations. Express α_c in terms of $f_0, \Delta T, T_0$.

(c) 该水瓶由线性热膨胀系数为 α 的材料制成。假设在参考温度 T_0 下，共振器的校准频率为 f_0。当环境温度升高至 $T = T_0 + \Delta T$（其中 $\Delta T \ll T_0$）时：

(ci) [2] 推导新共振频率 f 的表达式，用$f_0, \Delta T, T_0$ 和 α 表达。

(cii) [1] 确定一个“临界”热膨胀系数 α_c，使得在温度发生微小波动时，瓶子的频率保持不变。用 $f_0, \Delta T, T_0$ 表达 α_c。

~ End of Part 1 卷-1 完 ~