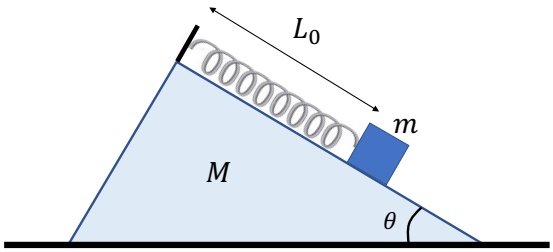
Pan Pearl River Delta Physics Olympiad 2022
2022 年泛珠三角及中华名校物理奥林匹克邀请赛
Sponsored by Institute for Advanced Study, HKUST
香港科技大学高等研究院赞助
Simplified Chinese Part-1 (Total 4 Problems, 40 Points) 简体版卷-1（共4题，40分）
(9:30 am – 12:00 pm, 14th May 2022)

Please fill in your final answers to all problems on the answer sheet.
请在答题纸上填上各题的最后答案。
At the end of the competition, please submit the answer sheet only. Question papers and working sheets will not be collected.
比赛结束时，请只交回答题纸，题目纸和草稿纸将不会收回。

1. [10 points] A wooden wedge of mass M is placed on a smooth table. A small wooden block of mass m is connected to the top of the wooden wedge with a spring of elastic constant k. Suppose the natural length of the spring is L_0, and there is no friction between two wooden blocks. Now the small block is released at rest at distance L_0 from the top of the wooden wedge (as shown in the picture) and slides down freely. Find
(a) [1pt] The equilibrium position of the small block m (i.e. the position where the acceleration of m vanishes) measured from the top of the wooden wedge.
(b) [2pt] The horizontal distance travelled by the wooden wedge when the small block m reaches its equilibrium position.
(c) [1pt] The oscillating amplitude of the small block m along the slope of the wedge.
(d) [6pt] The period of oscillation of the small block m along the slope of the wedge.

1. [10 分] 一块楔形木块，质量为 M ，被置放于一光滑之桌面上，另一质量为 m 之小木块以一弹力常数为 k 的弹簧系于楔形木块之顶端。设弹簧之原长为 L_0，且两木块之间无摩擦。今将小木块自离木块顶端 L_0 处静止释放（如图所示），让其自由滑下。试求



(a) [1 分] 小木块 m 之平衡位置(m 的加速度为零的位置)离楔形木块顶端之距离。

(b) [2 分] 小木块 m 到达其平衡位置时，楔形木块移动之水平距离。

(c) [1 分] 小木块 m 沿楔形木块斜面之振幅。

(d) [6 分] 小木块 m 来回振荡之周期。

2. [10 points] A system of 3 energy levels, $E_1 = 0, E_2 = \epsilon$, and $E_3 = 10\epsilon$ ($\epsilon > 0$) is populated by $N \gg 1$ particles at temperature T. The particles populate the energy levels according to the classical Boltzmann distribution law.
(a) [2pt] What is the average number of particles, N_3, with energy E_3?
(b) [2pt] What is the average energy of a particle at temperature T?
(c) [2pt] At sufficiently low temperature T_c, only energy levels E_1, E_2 are populated. Calculate the order of magnitude of the characteristic temperature T_c.
(d) [2pt] Calculate the molar specific heat at constant volume C_v at low temperature $k_B T \ll \epsilon$.
(e) [2pt] Calculate the molar specific heat at constant volume C_v at high temperature $k_B T \gg \epsilon$.

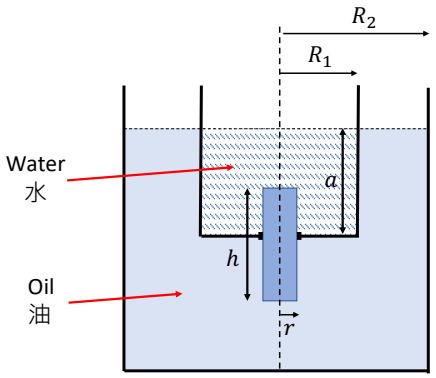
2. [10 分] 一个由 3 个能级 $E_1 = 0$、$E_2 = \epsilon$ 和 $E_3 = 10\epsilon$ ($\epsilon > 0$) 组成的系统在温度 T 下由 $N \gg 1$ 个粒子填充。这些粒子根据经典 Boltzmann 分布定律填充能级。
(a) [2 分] 具有能量 E_3 的粒子的平均数量 N_3 是多少？
(b) [2 分] 粒子在温度 T 下的平均能量是多少？
(c) [2 分] 在足够低的温度 T_c 下，系统仅填充 E_1、E_2 两个能级。计算特征温度 T_c 的数量级。
(d) [2 分] 计算低温 $k_B T \ll \epsilon$ 下等体积摩尔比热 C_v。
(e) [2 分] 计算高温 $k_B T \gg \epsilon$ 下等体积摩尔比热 C_v。

3. [10 points] One cylindrical vessel of radius R_1 is fixed inside another cylindrical vessel of radius R_2, as shown in the figure. In the bottom of the small vessel, there is a small hole with a bushing and a wooden cylinder of radius r and height $h = 21\text{cm}$ is inserted. The wooden cylinder can only move vertically relative to bushing without friction. Water is poured into the small vessel to a height of $a = 30\text{cm}$, and oil is poured into the large vessel to the same level. And the wooden cylinder is in equilibrium.
Given that the water density is $\rho_w = 1000\text{kg/m}^3$, the oil density is $\rho_o = 790\text{kg/m}^3$ and the wooden cylinder density is $\rho = 600\text{kg/m}^3$.

(a) [5 pt] Find the ratio of the length of the wooden cylinder immersed in the water to its total length h.
(b) [5 pt] Find the condition between ρ_w, ρ_o, r, R_1 and R_2 such that the equilibrium of the wooden cylinder is stable. (Hint: You need to consider the finite size effect of R_1, R_2 and r)

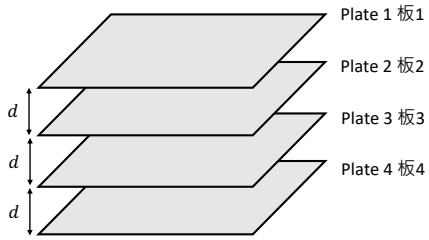
3. [10 分] 一个半径为 R_1 的圆柱容器固定在另一个半径为R_2 的圆柱容器内，如图所示。在小容器的底部有一个带衬套的小孔，插入半径为 r、高为 $h = 21\text{cm}$ 的木圆柱。木圆柱只能相对于衬套垂直移动而无摩擦。将水倒入小容器至 $a = 30\text{cm}$ 的高度，将油倒入大容器至同一高度。并且木圆柱处于平衡状态。
假设水的密度为 $\rho_w = 1000\text{kg/m}^3$，油的密度为 $\rho_o = 790\text{kg/m}^3$，木圆柱的密度为 $\rho = 600\text{kg/m}^3$。

(a) [5 分] 木圓柱在水中的部分與全長 h 的比例?
(b) [5 分] 找出 ρ_w, ρ_o, r, R_1 和 R_2 之间的条件，使得木圆柱的平衡是稳定的。（提示：您需要考虑 R_1、R_2 和 r 的有限尺寸效应）



4. [10 points] Four square conducting plates of area A are arranged at an even spacing d as shown in the diagram. (Assume that $A \gg d^2$)

4. [10 分] 如图所示，四块面积为 A 的方形导电板以等间距 d 排列。（假设$A \gg d^2$）



We perform the following steps to the system:
Step 1: Plate 1 and 4 are first connected to a voltage source of magnitude V_0, with plate 1 positive; Plate2 and 3 are connected with a wire.
Step 2: Remove the voltage source between plate 1 and 4.
Step 3: Remove the wire between plate 2 and 3.
Step 4: Finally, plate 1 and 4 are connected by a wire.

我们对系统执行以下步骤：

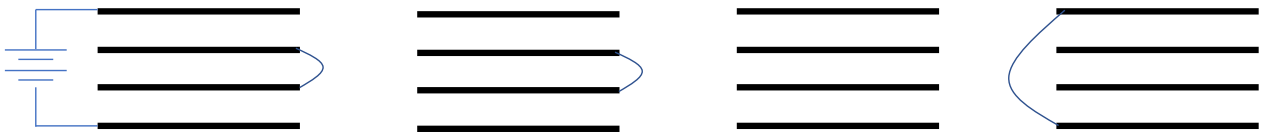
步骤 1：板 1 和 4 首先连接到幅度为V_0 的电压源，板 1 为正极；板 2 和板 3 用电线连接。

步骤 2：移除板 1 和板 4 之间的电压源。

步骤 3：拆下板 2 和 3 之间的电线。

步骤 4：最后，板 1 和 4 用电线连接。

The steps are summarized in the diagrams below.
下图总结了这些步骤。



Step 1 步骤1 | Step 2 步骤2 | Step 3 步骤3 | Step 4 步骤4

(a) [6pt] Find the potential difference ΔV_{12}, ΔV_{23} and ΔV_{34} at step 4, where $\Delta V_{ij} = V_i - V_j$ is the potential difference between plate i and j.

(a) [6 分] 求步骤 4 中的电位差 ΔV_{12}、ΔV_{23} 和 ΔV_{34}，其中 $\Delta V_{ij} = V_i - V_j$ 为板 i 和 j 之间的电位差。

(b) [4pt] What is the net electrostatic force acting on the plate 1 at step 4?

(b) [4 分] 求步骤 4 作用在板 1 上的净静电力是多少？

~ End of Part 1 卷-1 完 ~