2024 年第 20 届泛珠赛力学基础试 答题纸 (　月　日)

第 I 部分 选择题 (16×3 分)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
D	C	E	A	B	E	A	C	E	C	D	B	E	A	B	D

第 II 部分　简答题 (52 分)

17(a) 请写出主要步骤

By similar triangles, distance from center of small disk to O is l.

$$\tan\theta = \frac{a}{l}$$

$$= \frac{a}{\sqrt{24}a} = \frac{1}{\sqrt{24}}$$

$$\theta = \tan^{-1}\frac{1}{\sqrt{24}} = 0.201 \text{ rad}$$

$\theta =$

/4

17(b) 请写出主要步骤

Let R_1 be the radius of the circles traversed by small disc.

$$R_1 = \sqrt{l^2 + a^2}$$

$$= \sqrt{\left(\sqrt{24}\,a\right)^2 + a^2} = 5\,a$$

Let t be the time taken by the small disc to complete 1 revolution.
In this time, distance travelled by the disc = circumference of the disc = $2\pi a$.
Let θ' be the angle traversed by small disc in the circular trajectory in time t.

$$\theta' = \frac{\text{arc length}}{\text{radius of circle}}$$

$$= \frac{2\pi a}{5a} = \frac{2\pi}{5}$$

OR
Let R_2 be the radius of the circles traversed by large disc.

$$R_2 = \sqrt{(2l)^2 + (2a)^2} = 2\sqrt{\left(\sqrt{24}\,a\right)^2 + a^2} = 10\,a$$

Let t be the time taken by the large disc to complete 1 revolution. In this time, distance travelled by the disc = circumference of the disc = $4\pi a$.
Let θ' be the angle traversed by small disc in the circular trajectory in time t.

$$\theta' = \frac{\text{arc length}}{\text{radius of circle}}$$

$$= \frac{4\pi a}{10a} = \frac{2\pi}{5}$$

Angular speed along z-axis in time t:

$$\omega' = \frac{\theta'}{t}$$

/7

	$$= \frac{\frac{2\pi}{5}}{\frac{2\pi}{\omega}} = \frac{\omega}{5}$$ 组件质心绕 z 轴的角速率 = $\frac{\omega}{5}$	
17(c)	请写出主要步骤 Angular speed of the center of mass is the same: $$\omega' = \frac{\omega}{5}$$ Magnitude of angular momentum of the center of mass of the assembly about point *O*. $$\overline{L}_{COM,O} = \overline{r}_{COM} \times M\,\overline{V}_{COM}$$ $\overline{r}_{COM}$ = position vector from point *O* to COM $$r_{COM} = \frac{lm + (2l)4m}{5m}$$ $$= \frac{9l}{5}$$ As the assembly is rotating about *z*-axis, the center of mass of the assembly traverses a circle of radius *R*. $$R = r_{COM} \cos\theta$$ $$= \frac{9l}{5}\frac{\sqrt{24}}{5}$$ The linear velocity of the center of mass $$V_{COM} = R\omega'$$ $$= \frac{9l}{5}\frac{\sqrt{24}}{5}\frac{\omega}{5}$$ Therefore, the magnitude of the angular momentum of the COM of the assembly about point *O* is $$L_{COM,O} = \frac{9l}{5}5m\frac{9l}{5}\frac{\sqrt{24}}{5}\frac{\omega}{5}$$ $$= \frac{81\sqrt{24}}{125}m\omega l^2$$ Use $l = \sqrt{24}a$ $$L_{COM,O} = \frac{1944\sqrt{24}}{125}m\omega a^2$$ 绕点 O 的轨道角动量的大小 = $\frac{1944\sqrt{24}}{125}m\omega a^2$	/11

18(a)	请写出主要步骤 By momentum conservation $$mv + MV = mu + MU$$ $$m(v-u) = M(U-V)$$ By energy conservation $$\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \frac{1}{2}mu^2 + \frac{1}{2}MU^2$$ $$mv^2 - mu^2 = MU^2 - MV^2$$ $$m(v-u)(v+u) = M(U-V)(U+V)$$ Since the trivial unphysical solution $v = u$, $V = U$ is to be rejected, hence $$v + u = U + V$$ $$V = v + u - U$$ $$m(v-u) = M\big(U - (v+u-U)\big)$$ $$v = \frac{m-M}{m+M}u + \frac{2M}{m+M}U$$ $$V = \frac{m-M}{m+M}u + \frac{2M}{m+M}U + u - U$$ $$= \frac{2m}{m+M}u - \frac{m-M}{m+M}U$$ $$\begin{cases} v = \frac{m-M}{m+M}u + \frac{2M}{m+M}U \\ V = \frac{2m}{m+M}u - \frac{m-M}{m+M}U \end{cases}$$ (写出含 m,M,u 和 U 的表达式)	/4
18(b)	请写出主要步骤 $$\begin{cases} v_1 = -\frac{2M}{M+m}U \\ V_1 = \frac{M-m}{M+m}U \end{cases}$$ $$\tan\theta = \frac{\sqrt{m}v_1}{-\sqrt{M}V_1}$$ $$= \frac{\sqrt{m}}{\sqrt{M}}\frac{2M}{M-m} = \frac{2\sqrt{Mm}}{M-m}$$ $$\theta = \tan^{-1}\frac{2\sqrt{Mm}}{M-m}$$	/6

18(c)

请写出主要步骤

We have

Energy conservation:

$$mv_n^2 + MV_n^2 = m(-v_{n+1})^2 + MV_{n+1}^2$$
$$\left(\sqrt{m}v_n\right)^2 + \left(\sqrt{M}V_n\right)^2 = \left(\sqrt{m}(-v_{n+1})\right)^2 + \left(\sqrt{M}V_{n+1}\right)^2$$

Momentum conservation:

$$mv_n + MV_n = m(-v_{n+1}) + MV_{n+1}$$
$$\sqrt{m}\left(\sqrt{m}v_n\right) + \sqrt{M}\left(\sqrt{M}V_n\right) = \sqrt{m}\left(\sqrt{m}(-v_{n+1})\right) + \sqrt{M}\left(\sqrt{M}V_{n+1}\right)$$

By energy conservation, for all n, $\left(\sqrt{M}V_n, \sqrt{m}v_n\right)$ must lie on a circle with radius $-\sqrt{M}U$ centered at the origin.

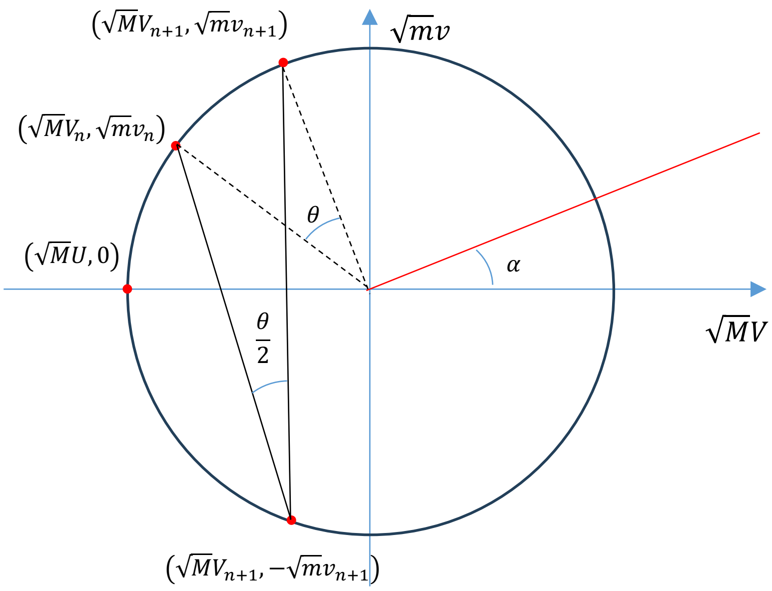
By momentum conservation,

$$\tan\frac{\theta}{2} = \frac{\sqrt{M}V_{n+1} - \sqrt{M}V_n}{\sqrt{m}v_n - \sqrt{m}(-v_{n+1})}$$

$$= \sqrt{\frac{m}{M}}$$

$$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}}$$

$$= \frac{2\sqrt{m/M}}{1 - m/M} = \frac{2\sqrt{Mm}}{M - m}$$



It can be seen that for consecutive collisions between the two blocks, the angular position increases by θ, and is $n\theta$ just before the n^{th} collision . Hence

$$\begin{pmatrix} \sqrt{m}v_n \\ \sqrt{M}V_n \end{pmatrix} = \begin{pmatrix} -\sqrt{M}U\sin n\theta \\ \sqrt{M}U\cos n\theta \end{pmatrix}$$

$$\begin{pmatrix} v_n \\ V_n \end{pmatrix} = \begin{pmatrix} -\sqrt{\frac{M}{m}}\sin n\theta \\ \cos n\theta \end{pmatrix}$$

/13

	$$\begin{cases} v_n = -\sqrt{\frac{M}{m}} U \sin\left(n \tan^{-1} \frac{2\sqrt{Mm}}{M-m}\right) \\ V_n = U \cos\left(n \tan^{-1} \frac{2\sqrt{Mm}}{M-m}\right) \end{cases}$$ (写出含 m,M和 U的表达式)	
18(d)	请写出主要证明步骤 Solve $V_n \geq v_n$ $$-\cos n\theta \geq \sqrt{\frac{M}{m}} \sin n\theta$$ $$\tan\theta = \frac{2\sqrt{Mm}}{M-m}$$ For $M \gg m$, the inequality can be satisfied for the first time when $n\theta = \pi$, i.e., $$n \sim \pi/\theta$$ $$N = 2n$$ $$\sim 2\pi/\theta$$ However, $$\theta \approx \tan\theta$$ $$\approx 2\sqrt{\frac{m}{M}}$$ Hence $$N \sim \frac{2\pi}{2\sqrt{\frac{m}{M}}} = \pi\sqrt{\frac{M}{m}}$$ 渐近线的斜率 = π	/7